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COMPARATIVE STUDY OF THE AXIAL AND AZIMUTHAL BUNCHING MECHANISM--ETC(U)

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Comparative Study of the Axial and Azimuthal Bunching Mechanisms in Electromagnetic Cyclotron Instabilities,

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A comparative study is presented of the axial and azimuthal bunching mechanisms of the electromagnetic electron cyclotron instability. Axial bunching can be described with a nonrelativistic treatment, but azimuthal bunching is relativistic in origin. As is well known, the axial bunching mechanism drives the Weibel-type instability while the azimuthal bunching mechanism drives the electron cyclotron maser instability. For an electron ensemble of cold helical trajectories, ~~we present~~ a unified physical interpretation of both instabilities. It is shown that the two bunching mechanisms

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are actually simultaneously present in either instability and compete with one another. As a result, the dominant mechanism determines the type of instability. A criterion for distinguishing the twc types of instabilities is derived. It is shown that the energy of the electrons plays an insignificant role in the criterion and, hence, should not be a factor in the justification of a nonrelativistic treatment. Regimes of validity of nonrelativistic models are defined, however. Applications to gyrotron experiments are discussed.

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COMPARATIVE STUDY OF THE AXIAL AND AZIMUTHAL BUNCHING MECHANISMS IN ELECTROMAGNETIC CYCLOTRON INSTABILITIES

I. INTRODUCTION

In the late 1950s, Twiss,[1] Schneider,[2] and Gaponov[3] independently proposed a radiation mechanism involving electrons gyrating in an external magnetic field. This phenomenon known as the electron cyclotron maser instability has been the subject of continuing activity.[4-29] It is the relativistic mass dependence of the electron cyclotron frequency which causes electrons to bunch azimuthally in their gyration orbits, and consequently to drive the instability. Recently, intense research activities to employ this interaction has occurred in the development of a new microwave source called the gyrotron or electron cyclotron maser. Results reported so far are impressive in terms of peak power[23] (1 GW at $\lambda = 4$ cm) and in terms of efficient production of large c.w. power[16,18] (12 kW at $\lambda = 2.8$ mm and 1.5 kW at $\lambda = 0.9$ mm). The high power, high efficiency, short wavelength radiation produced by gyrotrons can be employed, among other applications, to achieve local or global heating of tokamak plasmas.[30,31]

On the other hand, Weibel[32] and later authors[33-41] have investigated a different electromagnetic instability driven by the anisotropic velocity distribution of electrons. In contrast to the cyclotron maser instability, this instability is nonrelativistic in

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nature and will be referred to as the Weibel-type instability in the present paper. In the absence of wave-particle resonances, the basic mechanism of Weibel-type instability is axial electron bunching[42] in the direction of wave propagation, caused by the $\underset{\sim}{v}_\perp \times \underset{\sim}{B}_1$ Lorentz force, where $\underset{\sim}{v}_\perp$ is the electron velocity component perpendicular to the wave vector and $\underset{\sim}{B}_1$ is the wave magnetic field. In the presence of an electron thermal spread, the distribution of axial velocity will tend to spoil the phase synchrony discussed, thereby reducing the growth.

There are fundamental differences between the two instabilities. In addition to the qualitative difference in bunching mechanisms, the Weibel-type instability does not even require an external magnetic field, except to define an axis for the velocity space anisotropy. Yet, there are also similarities between the two instabilities. Both will take place for right-hand circularly polarized wave propagating along the external magnetic field and the unstable spectra both scale with the electron cyclotron frequency. From such similarities, one expects a close relationship between the two instabilities. In fact, there have been speculations among gyrotron researchers that the Weibel-type instability might also be exploited for microwave generation, or may even be responsible for some observations in gyrotron experiments.[4] From the literature on the two types of instabilities, one finds that investigations of these two subjects have evolved separately. Most studies concerning the Weibel-type instability have been based on non-relativistic models and thus do not include the cyclotron maser instability. On the other hand, most treatments on the cyclotron maser instability have concentrated on the azimuthal bunching mechanism. Hirshfield, et al[6]

briefly discussed the relative importance of the terms originating from the two mechanisms in their result. They found that the azimuthal bunching mechanism dominates when the wave phase velocity exceeds the speed of light and vice versa, a conclusion we will verify from a different point of view and elaborate further in the present treatment.

This study is motivated by recent interest in the cyclotron maser instability. Its purpose is to examine the two bunching mechanisms under a unified physical picture and thereby clarify the physical relation between the two types of instabilities. Our analysis will be based on the following electron distribution function commonly adopted for cyclotron maser studies,

$$f_o = \delta(p_\perp - p_{\perp o})\, \delta(p_z)/2\pi p_\perp \tag{1}$$

where $p_\perp, p_z$ are the transverse and axial momenta, respectively, $p_{\perp o}$ is a constant and $\delta(x)$ is the Dirac delta function. Eq. (1) is a realistic beam frame representation of the electron beams used in most gyrotron experiments. For such a cold distribution function, wave-particle resonances are absent. Our results are thus strictly valid for sufficiently monoenergetic electron distribution functions which can be approximated by Eq. (1).

We find from the simple model to be presented that there is indeed a strong coupling between the bunching mechanisms associated with the two instabilities. They are so related and inseparable that only a unified physical interpretation can fully describe the physical processes involved

in either instability. Except for the special cases of zero external magnetic field or infinite wavelength, the two mechanisms are always simultaneously present and compete rather than reinforce one another.

By comparing the two mechanisms, we obtain a criterion for determining the relative importance of the two mechanisms and identify the two types of instabilities as the fast-wave and slow-wave branches of the right-hand circularly polarized wave without actually solving the dispersion relation. From the comparison, it can also be seen that the two mechanisms interact in such a way that, for a given k_z, only one branch can be unstable. Another significant feature resulting from the comparison is that the (relativistic) azimuthal bunching mechanism may <u>dominate</u> at "nonrealativistic" electron energies, while the (nonrelativistic) axial bunching mechanism may <u>dominate</u> at "relativistic" electron energies.

In Section II we compare in detail the azimuthal and axial bunching mechanisms. In Section III we illustrate the conclusions of Section II with numerical examples. Section IV gives further discussions.

II. COMPARISON BETWEEN AXIAL AND AZIMUTHAL BUNCHING MECHANISMS

Consider an infinite and uniform plasma (density n) immersed in a uniform magnetic field ($B_o \underset{\sim}{e}_z$). For waves propagating along the magnetic field, it is well known that the electromagnetic and electrostatic effects are decoupled, resulting in purely electromagnetic ($\nabla \cdot \underset{\sim}{E} = 0$) or electrostatic ($\nabla x \underset{\sim}{E} = 0$) modes. The mode of interest here is the electromagnetic mode with right-hand polarization (i.e., electric field rotates in the same sense as electron gyration) and with wave frequency (or growth rate) comparable to the electron cyclotron frequency (or plasma frequency). In such a case, the plasma ions simply provide a charge neutralizing background and their dynamics can be neglected. For the case of a pure electron gas (such as in gyrotron experiments), we assume that the electron gas is sufficiently tenuous that its static space-charge electric field can be neglected (a good assumption for most gyrotron experiments).

Under these assumptions, the governing equations are the linearized relativistic Vlasov equation,

$$\frac{\partial}{\partial t} f_1 + \underset{\sim}{v} \cdot \frac{\partial}{\partial \underset{\sim}{x}} f_1 - \frac{e}{c} \underset{\sim}{v} x \underset{\sim}{B}_o \cdot \frac{\partial}{\partial \underset{\sim}{p}} f_1 = e(\underset{\sim}{E}_1 + \frac{1}{c} \underset{\sim}{v} x \underset{\sim}{B}_1) \cdot \frac{\partial}{\partial \underset{\sim}{p}} f_o \ , \qquad (2)$$

and the field equation

$$\nabla x \nabla x \underset{\sim}{E}_1 = - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \underset{\sim}{E}_1 - \frac{4\pi}{c^2} \frac{\partial}{\partial t} \underset{\sim}{J}_1 \ , \qquad (3)$$

where f_o and f_1 are, respectively, the equilibrium and perturbed distributions; $\underset{\sim}{E}_1$, $\underset{\sim}{B}_1$ are the wave electric and magnetic fields; and

$$\underset{\sim}{J}_1 = - e\int f_1 \, \underset{\sim}{v} d^3p \tag{4}$$

is the perturbed current.

The solutions to Eqs. (2) through (4) can be obtained by standard procedures.[43] Letting all quantities vary as $\exp(-i\omega t + ik_z z)$ and applying the method of characteristics, we obtain the relativistic dispersion relation for the right-hand circularly polarized wave,

$$\omega^2 - k_z{}^2c^2 = - \pi\omega_p^2 \int_0^\infty p_\perp dp_\perp \int_{-\infty}^\infty dp_z \frac{\left(\omega - \frac{k_z p_z}{\gamma m}\right) p_\perp \frac{\partial f_o}{\partial p_\perp} + \frac{k_z}{\gamma m} p_\perp^2 \frac{\partial f_o}{\partial p_z}}{\gamma\omega - k_z p_z/m - \Omega_e}, \tag{5}$$

where $\omega_p^2 = 4\pi ne^2/m$, $\Omega_e = eB_o/mc$, and

$$\gamma = \left(1 + p_\perp{}^2/m^2c^2 + p_z{}^2/m^2c^2\right)^{\frac{1}{2}} \tag{6}$$

Equation (5) can be integrated by parts to give

$$\omega^2 - k_z^2c^2 = 2\pi\omega_p^2 \int_0^\infty p_\perp dp_\perp \int_{-\infty}^\infty dp_z \, \frac{f_o}{\gamma} \left[\frac{\omega - k_z p_z/\gamma m}{\omega - k_z p_z/\gamma m - \Omega_e/\gamma} - \frac{p_\perp^2\left(\omega^2 - k_z^2c^2\right)}{2\gamma^2 m^2 c^2 \left(\omega - k_z p_z/\gamma m - \Omega_e/\gamma\right)^2} \right] \tag{7}$$

In carrying out the integration by parts over $p_\perp$ and p_z the integrand in Eq. (5) has been differentiated with respect to $p_\perp$ and p_z. The ω^2 term in the numerator of the second term on the right-hand side of Eq. (7) is the net result from differentiations of γ with respect to $p_\perp$ and p_z.

We may now compare Eq. (7) with the standard nonrelativistic version of the same dispersion relation,[44]

$$\omega^2 - k_z^2c^2 = 2\pi\omega_p^2 \int_0^\infty v_\perp dv_\perp \int_{-\infty}^\infty dv_z\ f_0 \left[\frac{\omega - k_zv_z}{\omega - k_zv_z - \Omega_e} + \frac{k_z^2v_\perp^2}{\left(\omega - k_zv_z - \Omega_e\right)^2}\right] . \tag{7'}$$

Equation (7) is invariant in form under Lorentz transforms and in the nonrelativistic limit ($c \rightarrow \infty$), it reduces to Eq. (7').

Comparing Eqs. (7) and (7'), one finds that the ω^2 term in Eq. (7), which originates from the relativistic mass factor γ, is absent from Eq. (7') as expected. Since this term is multiplied by $p_\perp^2/c^2$, it becomes small in the nonrelativistic limit. However, it is always an important term in the physical sense because it gives rise to the cyclotron maser instability.

For the distribution function given by Eq. (1), Eq. (7) reduces to

$$\omega^2 - k_z^2c^2 = \frac{\omega_p^2}{\gamma_0}\left[\frac{\omega}{\omega - \Omega_e/\gamma_0} - \frac{\beta_{\perp 0}^2\left(\omega^2 - k_z^2c^2\right)}{2\left(\omega - \Omega_e/\gamma_0\right)^2}\right] , \tag{8}$$

where $\gamma_o \equiv \left(1 + p_{\perp o}^2/m^2c^2\right)^{\frac{1}{2}}$ and $\beta_{\perp o} \equiv p_{\perp o}/\gamma_o mc$.

For the same distribution function in the nonrelativistic limit, Eq. (7') reduces to

$$\omega^2 - k_z^2c^2 = \omega_p^2 \left[\frac{\omega}{\omega - \Omega_e} + \frac{k_z^2 v_{\perp o}^2}{2(\omega - \Omega_e)^2}\right] , \tag{8'}$$

where $v_{\perp o} = \beta_{\perp o} c$.

Again we point out that the difference between Eq. (8) and Eq. (8') in the limit $\gamma_o \to 1$ is the presence of the ω^2 term on the right hand side of Eq. (8).

We will now seek a unified physical interpretation of the two instabilities. Consider an electron moving with velocity v_z in the wave fields which vary as $\exp(-i\omega t + ik_z z)$. Conventionally, one defines a Doppler shifted wave frequency $\omega_D \equiv \omega - k_z v_z$ as the effective wave frequency observed by the moving electron. To facilitate the physical interpretation of the present problem, we define instead a Doppler shifted cyclotron frequency

$$\Omega_D \equiv k_z v_z + \Omega_e/\gamma \tag{9}$$

as the effective cyclotron frequency of a moving electron observed by the propagating wave.

Figure 1 shows the instantaneous vector relationship of the wave fields ($\underset{\sim}{E}_1$ and $\underset{\sim}{B}_1$), the external magnetic field ($\underset{\sim}{B}_o$), the

positions (points 1 and 2) and perpendicular velocities ($\underset{\sim}{v}_{\perp}$) of two electrons located on the same vertical line (x=0) at the initial time t=0. The projection of the unperturbed electron orbit on the x-y plane is shown by the dashed circle. We have assumed that the wave propagates in the positive z-direction.

The instantaneous value of Ω_D at t=0 is

$$\Omega_D(0) = k_z v_z(0) + \Omega_e/\gamma(0) = \Omega_e/\gamma(0). \tag{10}$$

where we have let $v_z(0) = 0$ in accordance with Eq. (1).

After an infinitesimal time Δt, the Doppler shifted cyclotron frequency Ω_D will be

$$\Omega_D(\Delta t) = k_z v_z(\Delta t) + \Omega_e/\gamma(\Delta t) \tag{11}$$

Equation (11) minus Eq. (10) gives

$$\Delta\Omega_D = k_z \Delta v_z + \Omega_e \left[\frac{1}{\gamma(0) + \Delta\gamma} - \frac{1}{\gamma(0)}\right]$$

or $$\Delta\Omega_D \simeq k_z \Delta v_z - \Omega_e \Delta\gamma/\gamma^2(0) \tag{12}$$

where $\Delta\Omega_D \equiv \Omega_D(\Delta t) - \Omega_D(0)$, $\Delta v_z \equiv v_z(\Delta t)$, and $\Delta\gamma \equiv \gamma(\Delta t) - \gamma(0)$.

From the relativistic equation of motion,

$$m \frac{d}{dt} \gamma \underset{\sim}{v} = -e\underset{\sim}{E} - \frac{e}{c} \underset{\sim}{v} \times \underset{\sim}{B}_1$$

we obtain

$$\Delta v_z \underset{\sim}{e}_z \approx \frac{-e}{\gamma(0)mc} \underset{\sim}{v}_{\perp} \times \underset{\sim}{B}_1 \Delta t \tag{13}$$

and

$$\Delta\gamma \simeq \frac{-e}{mc^2} \underset{\sim}{v}_\perp \cdot \underset{\sim}{E}_1 \, \Delta t \quad . \tag{14}$$

Substituting the relation (Faraday's law),

$$\underset{\sim}{B}_1 = \frac{c}{\omega} \underset{\sim}{k} \times \underset{\sim}{E}_1 \tag{15}$$

into Eq. (13) to eliminate $\underset{\sim}{B}_1$, we obtain

$$\Delta v_z = - \frac{ek_z}{\gamma_o m\omega} (\underset{\sim}{v}_\perp \cdot \underset{\sim}{E}_1) \, \Delta t, \tag{16}$$

where we have let $\gamma(0) = \gamma_o$.

Substituting Eqs. (14) and (16) into Eq. (12) yields

$$\Delta\Omega_D = \frac{-ek_z^2}{\gamma_o m}\left(1 - \frac{\omega\Omega_e}{\gamma_o k^2 c^2}\right) \underset{\sim}{v}_\perp \cdot \underset{\sim}{E}_1 \quad \Delta t \tag{17}$$

From Eq. (17), we draw the following conclusions:

(i) Since $v_\perp$ for the two electrons are oppositely oriented, Eq. (17) shows that $\Delta\Omega_D$ for the two electrons will be of opposite sign. Consequently, if electron 1 slips (gains) in phase angle in the phase space of the Doppler shifted cyclotron oscillation (Ω_D) electron 2 will gain (slip) in phase angle. As a result, the two electrons tend to bunch toward each other. The first term on the RHS of Eq. (17) (due to Δv_z) results in axial bunching and the second term (due to $\Delta\gamma$) results in azimuthal bunching.

(ii) The two bunching mechanisms will be simultaneously present except for the special case $k_z = 0$ or $\Omega_e = 0$.

(iii) As will be shown later, ω and Ω_e are of the same sign. Hence the two terms on the RHS of Eq. (17) are of opposite sign, which implies that the two bunching mechanisms always combine in such a way as to <u>offset</u> one another.

(iv) The azimuthal bunching mechanism dominates if

$$\frac{\omega\Omega_e}{\gamma_o k_z^2 c^2} > 1, \tag{18}$$

and the axial bunching mechanism dominates if

$$\frac{\omega\Omega_e}{\gamma_o k_z^2 c^2} < 1. \tag{19}$$

The same arguments leading to these conclusions also hold for any electron pair in the same circle but on a vertical line with $x \neq o$ (see Fig. 1). Thus, the above conclusions apply to the system as a whole.

In comparing Eqs. (7) and (7'), we have shown that even in the limit $\gamma_o \to 1$, the relativistic and nonrelativistic versions of the dispersion relation of the right hand circularly polarized wave in an anisotropic plasma are qualitatively different. Here in the criterion of Eqs. (18) and (19), we again see that γ_o is not a decisive factor in determining the relative importance of the two bunching mechanisms. This confirms our statement in Section I that electron energy is not the criterion to determine whether a relativistic or nonrelativistic model should be used to treat the present problem. The proper criterion is given in Eqs. (18) and (19).

Energy exchange processes between waves and bunched electrons for the two instabilities can also be viewed in a unified way. Consider the phase space of the Doppler shifted cyclotron oscillation. If the electrons are uniformly distributed in the phase space, then clearly no net energy can be extracted from the electrons through their interactions

with the wave electric field. If the electrons are bunched in the phase space of Ω_D due to either the azimuthal or axial bunching mechanism, then net beam energy loss is possible provided the bunched electrons remain in the energy extracting phase of the wave electric field. This imposes the condition $\omega \approx \Omega_D$. For the distribution function of Eq. (1), the condition reduces to

$$\omega \approx \Omega_e/\gamma_0 \tag{20}$$

Substituting Eq. (20) into Eqs. (18) and (19), we find that

$$\frac{\omega^2}{k_z^2} > c^2 \tag{18'}$$

is the regime where azimuthal bunching dominates, and

$$\frac{\omega^2}{k_z^2} < c^2 . \tag{19'}$$

is the r[illegible]gime where axial bunching dominates.

[illegible] alternative forms, Eqs. (18') and (19'), of the criterion in Eqs[illegible]) and (19) provide a simple way for mode identification--the azimuth[illegible] bunching mechanism destabilizes the fast branch (vacuum mode) of the right-hand circularly polarized wave, while the axial bunching mechanism destabilizes the slow branch (whistler mode) of the same wave.

One notes that, because of the competitive nature of the two bunching mechanisms, only one branch can be unstable for a given k_z. Thus the criterion in Eqs. (18') and (19') also serves to distinguish the type of a given instability.

As a consistency check, we observe from Eq. (12) that for $k_z=0$, only the azimuthal bunching mechanism is present. Thus we expect an instability from Eq. (8) but not from Eq. (8'). The absence of instability in Eq. (8') for $k_z=0$ is obvious. To show that there is an instability in Eq. (8) for $k_z=0$, we neglect the first term on the right-hand side of Eq. (8). The equation is then readily solved to give

$$\omega \approx \frac{\Omega_e}{\gamma_o} + i\,\frac{\beta_{\perp o}\omega_p}{(2\gamma_o)^{1/2}} \quad . \tag{21}$$

This is the cyclotron maser instability in the limit $k_z=0$.

Substituting Eq. (21) into the right hand side of Eq. (8), we obtain the following condition for neglecting the first term,

$$\frac{\beta_{\perp o}\Omega_e}{(2\gamma_o)^{1/2}\,\omega_p} \gg 1 \quad . \tag{22}$$

In the limit $k_z \rightarrow \infty$, the axial bunching mechanism dominates [see Eq. (19)]. Then, both Eq. (8) and Eq. (8') should yield the same solution, as can be easily demonstrated. In this limit, ω is also given by Eq. (21), except that it is not subject to the restriction Eq. (22).

III. NUMERICAL EXAMPLES

In this section we present two representative numerical examples to illustrate the various points made in Section II. For reasons described below, we may distinguish two characteristic regimes according to the ratio of electron cyclotron frequency to plasma frequency. In regime I, $\omega_p > \Omega_e/\gamma_o$ while in regime II, $\omega_p < \Omega_e/\gamma_o$.

In regime I, the cutoff frequency (ω_p) of the fast mode exceeds the electron cyclotron frequency. As a result, condition (20) cannot be satisfied for the fast mode and only the slow mode can be unstable (Weibel-type instability). This is the regime in which nonrelativistic models give fairly good approximations.

Figures 2a and 2b are, respectively, solutions of Eqs. (8) and (8') (parameters specified in the caption). We have chosen a small value for γ_o so that the relativistic [Eq. (8)] and nonrelativistic [Eq. (8')] dispersion relations are different mainly because of the presence of the ω^2 on the right hand side of Eq. (8). Comparisons of the two figures show that the nonrelativistic model, which neglects the azimuthal bunching mecnanism in the Weibel-type instability, only slightly overestimates the growth rate. In fact, the only noticeable difference between the two figures is the threshold k_z for the instability. The example given in Fig. 2 is typical of the entire range of Ω_e in this regime.

In regime II, both types of instabilities exist. All the conclusions reached in Section II are exhibited in this regime. Figures 3a and 3b are, respectively, solutions of Eq. (8) and (8') (parameters specified in the caption). These two figures are typical of the entire [illegible] To demonstrate the point that relativistic effect may be dom[illegible] relativistic energies, we have chosen a small value for γ_0 (corresponding to 10 keV). The dominance of relativistic effects results in the destabilization of the fast mode (dashed lines in Fig. 3a). The competitive nature of the two bunching mechanisms is apparent in Fig. 3a. We observe that the growth rates drop sharply when $\omega/k_z \approx c$, where the two mechanisms nearly cancel each other. For a given k_z, there is at most one unstable branch, as has been expected from previous considerations. Comparing Fig. 3a (relativistic model) with Fig. 3b (nonrelativistic model), we find some striking differences between the two models. First, the maser instability is absent from the nonrelativistic model. Second, the nonrelativistic model substantially overestimates the growth rate in the region $\omega/k_z \lesssim c$, where the relativistic bunching mechanism is also important. For larger values of k_z ($\omega/k_z \ll c$), the nonrelativistic model begins to yield more accurate results. Note that for smaller values of k_z, the growth rate for the slow wave mode drops to zero in the nonrelativistic model. This is, of course, not due to cancellation from the relativistic bunching mechanism. Rather, it is because condition (20) is not satisfied for small values of k_z.

IV. DISCUSSION

For the distribution function chosen [Eq. (1)], we have shown that a nonrelativistic treatment of the electromagnetic electron cyclotron instabilities is essentially valid in the regime(s) $\Omega_e/\gamma_o\omega_p < 1$ and/or $\omega/k_z << c$, but becomes very inaccurate in the regime $\Omega_e/\gamma_o\omega_p > 1$ and $\omega/k_z \lesssim c$. In the regime $\Omega_e/\gamma_o\omega_p > 1$ and $\omega/k_z > c$, nonrelativistic models are invalid because they completely miss the cyclotron maser instability. Generalization of the present results to distribution functions with large thermal spread can only be qualitative because of the neglect of wave-particle resonances in our model. The principal application of the present theory appears to be in the area of gyrotron research. As noted earlier, Eq. (1) is a realistic (beam frame) representation of the beam used in most gyrotron experiments. The physical relation between the axial and azimuthal bunching mechanisms has generally been overlooked in gyrotron studies. Clarification of this relationship may thus serve a useful purpose in identifying and predicting the type of instabilities in gyrotron experiments. The fact that only one type of instability can be present for a given wavelength makes mode identification less ambiguous. For example, the fast wave generated in the experiment of Chou and Pantell[4] has been attributed to axial bunching. In the light of the present study, it appears that the azimuthal bunching mechanism should have been responsible.

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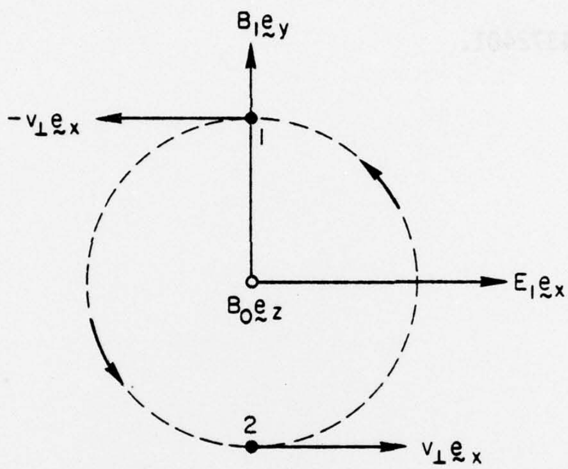

Fig. 1 — Instantaneous (t = 0) vector relationship of the wave fields (E_1 and B_1), the external magnetic field (B_0), the position (points 1 and 2) and perpendicular velocities ($v_\perp$) of two electrons. The projection of the unperturbed electron orbit on the x-y plane is shown by the dashed circle, the center of which is taken to be the origin of the Cartesian coordinate system. The positive z-axis points toward the reader.

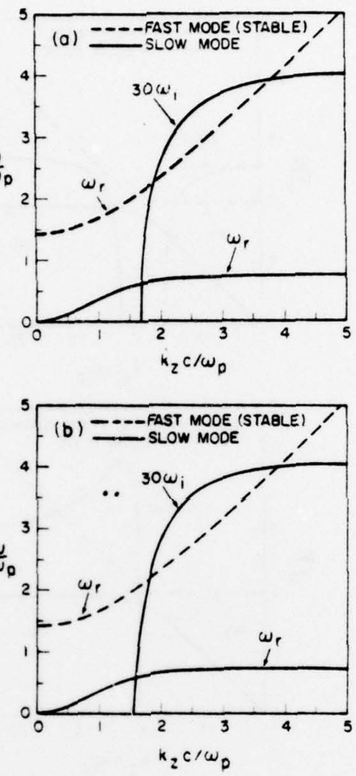

Fig. 2 — (a) ω versus k_z calculated from the relativistic dispersion relation, Eq. (8), for $\gamma_0 = 1.02$ and $\omega_e/\gamma_0\omega_p = 0.8$; (b) ω versus k_z calculated from the nonrelativistic dispersion relation, Eq. (8′), for $\Omega_e/\omega_p = 0.8$

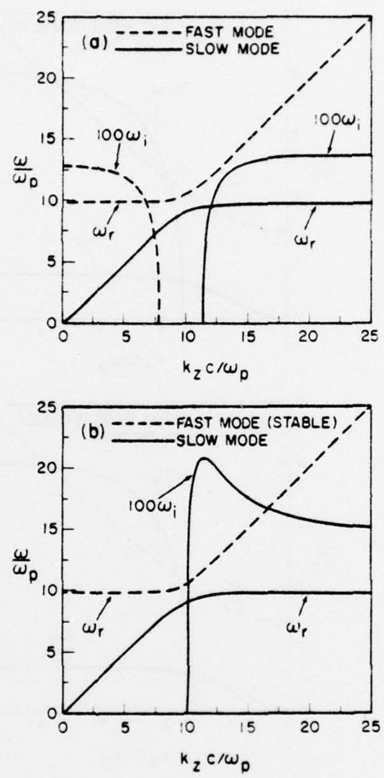

Fig. 3 — (a) ω versus k_z calculated from Eq. (8) for γ_0 = 1.02 and $\omega_e/\gamma_0\omega_p$ = 10; (b) ω versus k_z calculated from Eq. (8′) for ω/ω_p = 10

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